Complex Fractions

(1) A **complex fraction** is a fraction that contains a fraction. We have two techniques for simplifying complex fractions – one method always works and the other only works for what are often referred to as single nested complex fractions. To get at the idea of nesting, let's call $\dfrac{3+\frac{1}{x}}{1-\frac{1}{x^2}}$ the fraction Big F. Then, the fractions $\dfrac{1}{x}$ and $\dfrac{1}{x^2}$ are inside of Big F. This is an example of a **single nested** complex fraction – each "little" fraction sits inside only one other fraction, namely Big F. On the other hand, let's call $\dfrac{2+\frac{1}{x^2}}{1-\frac{1}{1+\frac{1}{x}}}$ the fraction Big D. Then, the fraction $\dfrac{1}{x^2}$ is inside of Big D (single nested), but the fraction $\dfrac{1}{x}$ is inside of $\dfrac{1}{1+\frac{1}{x}}$ which itself is inside of Big D. This is an example of a **multiple nested** complex fraction – at least one of the "little" fractions sits inside of more than one bigger fraction. Another way of defining multiple nested is to say it is a complex fraction which itself contains another complex fraction.

(2) The first method we'll discuss is the **LCD method**. It ONLY WORKS ON SINGLE NESTED COMPLEX FRACTIONS. All we do is *multiply numerator and denominator of the complex fraction by the LCD of all the "little" fractions*. For example:

(a) $\dfrac{3+\frac{1}{x}}{1-\frac{1}{x^2}}$ LCD$\left(\dfrac{1}{x},\dfrac{1}{x^2}\right)=x^2$, so

$$\frac{3+\frac{1}{x}}{1-\frac{1}{x^2}}\cdot\frac{x^2}{x^2}=\frac{\left(3+\frac{1}{x}\right)x^2}{\left(1-\frac{1}{x^2}\right)x^2}$$ now distribute the x^2

$$\frac{3x^2+x}{x^2-1}$$ note $\dfrac{1}{x}\cdot x^2=x$ and $\dfrac{1}{x^2}\cdot x^2=1$

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(b) $\dfrac{xy^{-1} + x^{-1}}{x^{-1} - 1}$ negative exponents with addition $\Rightarrow$ flip first

OoO 1b requires treating num. and denom. separately $\Rightarrow$ flipped things stay where they came from

$$\frac{\frac{x}{y} + \frac{1}{x}}{\frac{1}{x} - 1}$$

note $xy^{-1} = \dfrac{x}{y}$, NOT $\dfrac{1}{xy}$!!!!

LCD$\left(\dfrac{x}{y}, \dfrac{1}{x}\right) = xy$

$$\frac{\frac{x}{y} + \frac{1}{x}}{\frac{1}{x} - 1} \cdot \frac{xy}{xy} = \frac{\left(\frac{x}{y} + \frac{1}{x}\right)xy}{\left(\frac{1}{x} - 1\right)xy}$$ distribute

$$\frac{x^2 + y}{y - xy}$$

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(c) $\dfrac{a^{-1} + b^{-1}}{(a+b)^{-1}}$ neg. exponents with addition $\Rightarrow$ flip first

OoO 1b as above $\Rightarrow$ flipped things stay put
remember NEVER put a power through a sum!!!!!!

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a+b}}$$

LCD$\left(\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{a+b}\right) = ab(a+b)$

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a+b}} \cdot \frac{ab(a+b)}{ab(a+b)} = \frac{\left(\frac{1}{a} + \frac{1}{b}\right)ab(a+b)}{\left(\frac{1}{a+b}\right)ab(a+b)}$$ distribute

$$\frac{b(a+b) + a(a+b)}{ab}$$ factor $(a+b)$ out of numerator

$$\frac{(b+a)(a+b)}{ab} = \frac{(a+b)^2}{ab}$$ and make pretty at the end

(We'll do this last example in a cleaner way below using the other method.)

(3) The second method is the **Order of Operations method** which works on <u>all</u> complex fractions. Here we simply, very carefully follow the OoO to create a single simple fraction divided by another single simple fraction and then invert and multiply. For example:

(a) $\dfrac{3+\dfrac{1}{x}}{1-\dfrac{1}{x^2}}$ OoO 1b ⇒ treat num. and denom. separately

fraction + whole thing ⇒ add them up

$\dfrac{\dfrac{3x+1}{x}}{\dfrac{x^2-1}{x^2}}$ invert and multiply

$\dfrac{3x+1}{\not{x}}\cdot\dfrac{x^{\not{2}}}{x^2-1}=\dfrac{(3x+1)x}{x^2-1}$ making small before making big

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(b) $\dfrac{xy^{-1}+x^{-1}}{x^{-1}-1}$ neg. exponents with addition ⇒ flip first

OoO 1b ⇒ flipped things stay put

$\dfrac{\dfrac{x}{y}+\dfrac{1}{x}}{\dfrac{1}{x}-1}=\dfrac{\dfrac{x^2+y}{xy}}{\dfrac{1-x}{x}}$ add the terms in both num. and denom.

invert and multiply

$\dfrac{x^2+y}{\not{x}y}\cdot\dfrac{\not{x}}{1-x}=\dfrac{x^2+y}{y(1-x)}$ making small before making big

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(c) $\dfrac{a^{-1}+b^{-1}}{(a+b)^{-1}}$ neg. exponents with addition ⇒ flip first

OoO 1b ⇒ flipped things stay put
NEVER put a power through a sum!!!!

$\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a+b}}=\dfrac{\dfrac{b+a}{ab}}{\dfrac{1}{a+b}}$ add terms, then invert and multiply

$\dfrac{b+a}{ab}\cdot\dfrac{a+b}{1}=\dfrac{(a+b)^2}{ab}$ making pretty

(4) Which method you use is up to you for single nested problems. In this course we'll only see single nested complex fractions. However, those of you going on may run across double nested ones so you need to remember the Order of Operations method.