DRT Handout

Distance, rate, time problems in algebra usually contain some common key features which you must understand. Moreover, if you do understand these features, you should have a better chance at attacking non-standard d-r-t problems that may arise. To explore these features we will refer to the quantities {distance, rate and time} as Quantities (with a capital Q). Listed below are the common features you will encounter.

(A) Two objects are in motion or one object is in motion in two different ways. These are the two **Scenarios** of the problem.
(B) The unknown you are specifically asked to find is either a distance, a rate or a time. Whichever it is becomes the **Variable Quantity (VQ)**. You must then introduce an appropriate variable and declare it in a LEGEND along with the units with which it is measured. Then the VQ in both Scenarios is expressed in terms of this variable.
(C) Specific values for one of the Quantities will be given for both Scenarios. This will be called the **Constant Quantity (CQ)**.
(D) The quantity that is now left is the **Expression Quantity (EQ)**. Using ***d = rt*** we write an Expression for this Quantity in both Scenarios in terms of the Variable and Constant Quantities.
(E) There will be a size **Relation** between the two values of the Expression Quantity in the two Scenarios which will naturally lead to an equation involving them.

Example 1: Car A leaves Washington for Baltimore (**55** miles away) at the same time Car B leaves Baltimore for Washington on the same road. If Car A averages **60** mph and Car B averages **50** mph, set up the equation to find how long before they pass each other?

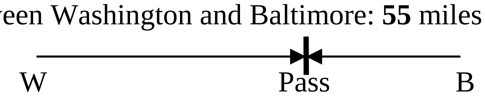
Solution: (A) The two *Scenarios* are (1) Car A going from Wash. to Balt. and (2) Car B going from Balt. to Wash.

(B) You are asked about the time until they pass each other. So Time is the *Variable Quantity*, and we'll use the time in Scenario (1) for the variable: **Let *H* = number of hours Car A travels before it passes Car B**. Then the number of hours Car B travels before it passes Car A is also ***H***.

(C) The *Constant Quantity* is the rate since we are told specifically Car A's rate is **60** mph and Car B's rate is **50** mph.

(D) The *Expression Quantity* (the only one left) must be Distance (***d = rt***). So for Car A, distance **= (60** mph**)(*H*** hr**) = 60*H*** miles; for Car B, distance **= (50** mph**)(*H*** hr**) = 50*H*** miles.

(E) The *Relation* existing between the two values of the Expression Quantity is that they must add up to the total distance between Washington and Baltimore: **55** miles



60*H* miles + **50*H*** miles = **55** miles

Therefore, the answer is:

{ Let ***H*** = number of hours Car A travels before it passes Car B
60*H* + 50*H* = 55. }

Note that the addition on the left is legal since they are both miles, and the equation makes sense since we have miles on both sides. ∵

Example 2: It takes Peter **3** hours to drive on the expressway from his home to his resort cabin. Had he taken the scenic route, which is **10** miles longer, it would have taken him **5** hours because his average speed would be **18** mph less. Set up the equation to find his expressway average speed.

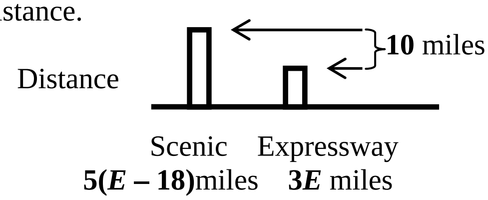
Solution: (A) The two *Scenarios* are going: (1) via the expressway and (2) via the scenic route.

(B) The requested quantity is the expressway rate, so rate is the *Variable Quantity*. **Let E = average speed in mph on the expressway route.** The corresponding Quantity in the other Scenario is the rate on the scenic route which we're told is **18** mph less, so the scenic route rate is ($E - 18$) mph. Note the subtraction is legal since they are both measured in mph.

(C) The *Constant Quantity* is time: **3** hours by expressway and **5** hours by the scenic route.

(D) The *Expression Quantity*, therefore, must be distance (it's the only one left). Since $d = rt$ we have the expressway distance is (E mph)(**3** hr) = $3E$ miles; and the scenic route distance is [($E - 18$) mph](**5** hr) = $5(E - 18)$ miles.

(E) The distance *Relation* is given by the following: the scenic distance is (i.e., =) **10** miles more than (i.e., +) the expressway distance.



So the answer is:

$$\left\{ \begin{array}{l} \text{Let } E = \text{average speed in mph on the expressway route} \\ 5(E-18) = 3E + 10 \end{array} \right\}$$

Note the addition on the right is legal since they are both miles, and the equation makes sense since we have miles on both sides. ∵

Example 3: Tom, at a leisurely **4** mph pace, walked two more miles than his brother, Peter, did who walks at a brisk **5** mph pace. However, Tom did walk one hour longer than Peter. Set up the equation to find how far Tom walked.

Solution: (A) *Scenarios*: (1) Tom's walk; (2) Peter's walk.

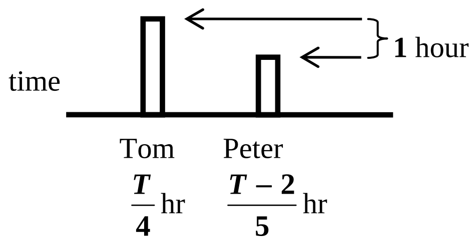
(B) *VQ*: It is distance, so **let T = distance in miles that Tom walked.** The corresponding distance value in the other Scenario is $T - 2$ = distance in miles that Peter walked.

(C) *CQ*: It is rate: Tom's rate (**4** mph) and Peter's rate (**5** mph).

(D) *EQ*: The one that is left is time ($t = \frac{d}{r}$). Tom's time = $\frac{\text{distance}}{\text{rate}} = \frac{T \text{ miles}}{4 \text{ mph}} = \frac{T}{4}$ hours;

Peter's time = $\frac{\text{distance}}{\text{rate}} = \frac{(T-2) \text{ miles}}{5 \text{ mph}} = \frac{T-2}{5}$ hours.

(E) *Relation*:



So the answer is:

$$\left\{ \begin{array}{l} \text{Let } T = \text{distance in miles Tom walked} \\ \dfrac{T}{4} = \dfrac{T-2}{5} + 1 \end{array} \right\}$$

Note again that the units work: hours + hours on the right and finally hours = hours. ∵

Boat and Plane Type Problems

The rate at which a boat is propelled in still water (e.g., a lake) is called the **still water rate** of the boat. When the boat is moving in a river with a current, its **ground speed** (speed as measured by an observer on the ground) is found by combining the still water speed with the rate of the current in the following way: if the boat is moving UPSTREAM (against the current), then the *ground speed = the still water speed MINUS the rate of the current*; if the boat is moving DOWNSTREAM (with the current), then the *ground speed = the still water speed PLUS the rate of the current*. This is due to the fact that moving against the current uses up some still water speed just to keep from being pushed backwards, and moving with the current generates movement of the boat without any effort on its part which is then an addition to the still water rate.

The above also applies to planes in flight going either against or with the wind. The speed of a plane flying in no wind is called the **air speed**. The ground speed is then found by either subtracting the wind speed from the air speed if flying against the wind or adding the wind speed to the air speed if flying with the wind.

Example 4: A boat that can travel **8** mph in still water can travel **36** miles upstream in twice the time it can travel **30** miles downstream. Set up the equation to find the rate of the current.

Solution: (A) *Scenarios*: (1) upstream travel; (2) downstream travel.

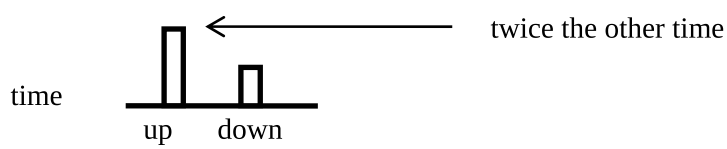
(B) *VQ*: We are asked to find the rate of the current, so the VQ is rate. **Let *C* = rate in mph of the current.** Then the rate in each Scenario is described using ***C***: Upstream rate = **8** mph **– C** mph **= (8 – *C*)** mph; Downstream rate = **8** mph **+ *C*** mph **= (8 + *C*)** mph.

(C) *CQ*: The upstream distance (**36** miles) and the downstream distance (**30** miles).

(D) *EQ*: The one left is time. So, using $\boldsymbol{t = \frac{d}{r}}$, the upstream time =

$\frac{\text{distance}}{\text{rate}} = \frac{\mathbf{36}\text{ miles}}{(\mathbf{8}-\boldsymbol{C})\text{ mph}} = \frac{\mathbf{36}}{\mathbf{8}-\boldsymbol{C}}\text{ hours}$; and the downstream time = $\frac{\mathbf{30}\text{ miles}}{(\mathbf{8}+\boldsymbol{C})\text{ mph}} = \frac{\mathbf{30}}{\mathbf{8}+\boldsymbol{C}}\text{ hours}$.

(E) *Relation*:



$\frac{\mathbf{36}}{\mathbf{8}-\boldsymbol{C}}$ hr $\qquad \frac{\mathbf{30}}{\mathbf{8}+\boldsymbol{C}}$ hr

So the answer is:

$$\left\{\begin{array}{l}\text{Let } \boldsymbol{C} = \text{rate in mph of the current.}\\ \dfrac{\mathbf{36}}{\mathbf{8}-\boldsymbol{C}} = 2\left(\dfrac{\mathbf{30}}{\mathbf{8}+\boldsymbol{C}}\right)\end{array}\right\}$$

Note both sides are hours so the equation makes sense. ∵

Before the test be sure you compare the DRT problems here with Example 2 on page 27 of The Fundamentals. The problem there involves distance, rate and time, but it is a very different type of problem. In that Example you are asked to find the average speed (rate) for the *entire* trip, not for one of the two Scenarios; hence you are working with the basic definition of average speed (i.e., total distance divided by total time). Don't get confused on a test if you are confronted with this type of problem.

Problems:

1. Two cars pass each other on the highway. One is going **50** mph due East and the other is going **55** mph due West. Set up the equation to determine how long before they are **300** miles apart.

2. A jet flew across the U.S. at an average speed of **500** mph. It then continued across the Atlantic at an average speed of **530** mph. If both parts of the trip took the same time and the total distance flown was **14,400** miles, set up the equation to determine how long the entire trip took.

3. Carl leaves home on his bicycle at 9:30am. His sister, Karen, who can ride **3** mph faster than Carl, leaves home at 10:30am to catch up with him. If they use the same route and Karen rides $\mathbf{13\frac{1}{3}}$ miles before catching Carl, set up the equation to find Karen's rate.

4. Sue drove at a steady speed for two hours. She then ran into heavy traffic which reduced her speed by **25** mph for the rest of the trip. Set up the equation to find her speed on the first part of the trip if the whole **150** mile drive took her **3** hours.

5. Tom runs **10** meters per second and Al runs **6** meters per second. If Al has a **50** meter head start, set up the equation to determine how long it takes Tom to catch Al.

6. A commuter takes an hour to get to work. Part of the trip is on the Metra train averaging **65** mph, and the rest is on a bus averaging **25** mph. Set up the equation to determine how far she rides on the bus if the entire distance traveled to work is **60** miles.

7. A **660** mile trip via ship and train takes **20** hours. If the ship averages **32** mph and the train averages **50** mph, set up the equation to find how long the ship part of the trip takes.

8. After riding at a steady rate for **40** miles, a bicyclist had a flat tire and walked **5** miles to a repair shop. The cycling rate was **4** times faster than her walking rate. If the entire trip took **5** hours, set up the equation to find the riding rate of the cyclist.

9. Two cities are **240** km apart. If the average speed of a train going from one to the other were to be increased by **6** km per hour, the train would arrive an hour and **20** minutes earlier. Set up the equation to find the normal average speed of this train.

10. A boat can travel **42** km downstream in **2** hours less time than it can travel **50** km upstream. If the rate of the river is **2** km/hr, set up the equation to find the still water rate of the boat.

11. At noon two planes take off and fly with an average air speed of **66** mph. One heads due North, into the wind, and the other heads due South, with the wind at its tail. At 3:00pm the North bound plane lands at city A. Two hours later the South bound plane lands at city B. If cities A and B are **536** miles apart, set up the equation to find the wind speed.

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1. Let t = time in hours of travel. $\mathbf{50t + 55t = 300}$. {miles + miles = miles}
2. Let t = time in hours of total trip. $\mathbf{500(t/2)+530(t/2) = 14400}$. {miles + miles = miles}
3. Let K = Karen's rate in mph. $\dfrac{40}{3(K-3)} = \dfrac{40}{3K} + 1$. {hours = hours + hours}
4. Let F = speed in mph on the first part of trip. $\mathbf{2F + 1(F - 25) = 150}$. {miles + miles = miles}
5. Let T = time in seconds they ran. $\mathbf{10T = 6T + 50}$. {meters = meters + meters}
6. Let B = distance in miles on bus. $\dfrac{60-B}{65} + \dfrac{B}{25} = 1$. {hours + hours = hours}
7. Let S = time in hours on ship. $\mathbf{32S + 50(20 - S) = 660}$. {miles + miles = miles}
8. Let C = cycling rate in mph. $\dfrac{40}{C} + \dfrac{20}{C} = 5$. {hours + hours = hours}
9. Let A = normal rate in km per hour of the train. $\dfrac{240}{A} = \dfrac{240}{A+6} + \dfrac{4}{3}$. {hours = hours + hours}
10. Let S = still water rate in km/hr of the boat. $\dfrac{42}{S+2} + 2 = \dfrac{50}{S-2}$. {hours + hours = hours}
11. Let W = wind speed in mph. $\mathbf{3(66 - W) + 5(66+W) = 536}$. {miles + miles = miles}