Fractional Equations

(1) The examples below are **rational equations**. However, the technique covered here applies to any equation that contains any type of fraction, not just a quotient of polynomials. Therefore, I'll refer to any equation containing a fraction as a **fractional equation**.

(2) To solve all fractional equations, the first step is to:

> *Multiply both sides by the LCD of all the fractions in order to get rid of all the fractions.*

ZZZZZ A word of caution which I should have mentioned above. The only exception to this as the first step is if there is a complex fraction anywhere in the equation. In that case, **simplify all complex fractions first**.

(3) Some examples.

(a) $\frac{1}{9}-\frac{1}{x}=\frac{1}{6}$ LCD $\left(\frac{1}{9},\frac{1}{x},\frac{1}{6}\right)=18x$

$18x\left(\frac{1}{9}-\frac{1}{x}\right)=18x\left(\frac{1}{6}\right)$ multiply

$2x-18=3x$ solve this linear equation

$x=18$

(b) $2-\frac{1}{x-2}=\frac{x-3}{x-2}$ LCD=$x-2$

$(x-2)\left(2-\frac{1}{x-2}\right)=(x-2)\left(\frac{x-3}{x-2}\right)$

$2(x-2)-1=x-3$ solve this linear equation

$\vdots$

$x=2$

ZZZZZ Opps!!! The original equation contained the expressions $\frac{1}{x-2}$ and $\frac{x-3}{x-2}$. This means that x is NEVER allowed to equal **2** when working with these expressions since it would create a **0** denominator. But $x = 2$ is the "solution" we got above. Technically, $x = 2$ is the solution to the linear equation $2(x-2)-1=x-3$, but it can't be a solution to the original equation because it is not in the domain for the expressions there. So we have to throw $x = 2$ away, and, hence, we are left with no values of x that satisfy the original equation. Thus the solution set is $\varnothing$.

(4) This last example introduces something new to equation solving – the necessity in certain situations to check your proposed solution. In the past, textbooks and teachers have encouraged you to check your answers, but this was done mainly as a way to check the accuracy of your work so you could catch a mistake you might have made. It was an optional activity if you had the time. Here, however, the checking is a different story – it is part of the solving process. Actually you don't have to plug your solution in and physically show the two sides of the resulting equation are in fact equal. All that is really needed is to **check to see if any of your proposed solutions create a 0 denominator anywhere in the original equation**. If it does, throw it away. For now, it will be an all or nothing situation: you will get one proposed solution, and either it will or it won't create a **0** denominator.