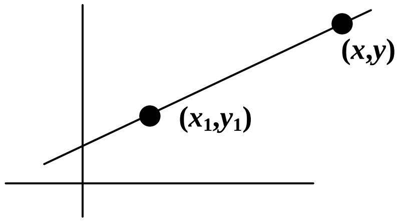
Lines

We know the standard form for a straight line: ***Ax* + *By* = *C***. We'll now derive other forms that are more useful. *We essentially need two things to create an equation for a line:* ***its slope and a point on it***. So, let's suppose we're given the slope, ***m***, of a line and a point (x_1, y_1) on it. (We sometimes say the line passes through the point (x_1, y_1)). We want an equation which, by the Fundamental Theorem of Analytic Geometry, will allow us to determine if any given point **(*x*,*y*)** sits on this line or not. So let's picture this arbitrary point on the line somewhere.



Since, both (x_1,y_1) and (x,y) sit on this line and since the slope, ***m***, is independent of the points used to calculate it, we must get the value **m** if we use these two points to calculate the slope: $m = \frac{y - y_1}{x - x_1}$. Multiplying both sides by $x - x_1$ gives the

> **Point-Slope Form of a Line:**
> $y - y_1 = m(x - x_1)$

It is that simple. Given the slope and a point, we just plug them into this equation.

Example 1: Find an equation for the line through **(5,-4)** having slope **2/3**.

Solution: $y - (-4) = \frac{2}{3}(x - 5)$. ∵

Yes, that's right, we're done. What could be easier? Of course, you might want to make this equation look prettier, but that is your decision. As far as I'm concerned, we're done.

Example 2: Find an equation for the line containing $(2, -5)$ and $(3,7)$.

Solution: Oops – no slope given. But, given two points we can always find the slope:

$$m = \frac{-5 - 7}{2 - 3} = \frac{-12}{-1} = 12.$$

Now we can choose either of the given points to get an equation:

$y - (-5) = 12(x - 2)$ using the first point, ***OR***

$y - 7 = 12(x - 3)$ using the second point. ∵

Now, you might ask "Can I really get two different answers to the question?" Well, yes and no. You get two different *looking* answers here, but they still describe the same line which you can see by massaging them into standard form: $12x - y = 29$. Some books require this form for answers, but I don't. Any form is acceptable.

Let's look more carefully at the two answers given in Example 2 above:

(a) $y - (-5) = 12(x - 2)$

(b) $y - 7 = 12(x - 3)$

Solving each of these for ***y*** gives:

(a) $y = 12x - 29$

(b) $y = 12x - 29$

Notice they are the same. This is no accident. This will always happen to any collection of equations for the same line. If we solve for ***y***, we will always get the same equation. This form of the equation gives us two important pieces of information. First, we note that the coefficient of ***x*** is just the slope of the line. Second, if we set $x = 0$, we get in this example $y = -29$. So the ordered pair $(0, -29)$ lies on the line. But any point with a zero ***x*** coordinate is on the ***y*** axis. So the point $(0, -29)$ is the ***y***-intercept of this line. This is the second piece of information we always get: the constant is the ***y***-intercept of the line. These observations lead to a third form of a line:

Slope-Intercept Form of a Line:
$$y = mx + b$$

If we massage any equation for a line into this form, we automatically know its slope (***m***) and its ***y***-intercept (***b***). For example, consider

$2x + 3y = 7$ subtract $2x$ from both sides

$3y = -2x + 7$ divide through by 3

$$y = \frac{-2}{3}x + \frac{7}{3}$$

Thus, the slope is $\frac{-2}{3}$ and the ***y***-intercept is $\frac{7}{3}$.