Mixtures

Non-Value Type:

Let us assume in the following that each mixture is composed of two ingredients: the **key** ingredient and the **rest**, which combine to form a **total** amount. To say we have a **40%** "key ingredient" mixture means that **40%** of (i.e., times) the Total amount is how much of the key ingredient we have, and **100% – 40% = 60%** of the Total amount is how much of "the rest" we have.

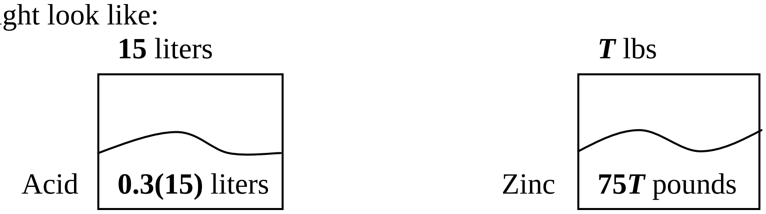
Example 1: (a) How much of **15** liters of a **30%** acid solution is pure acid? (b) If we add **3** liters of acid to this, what is the acid percentage in the new solution?

Solution: (a) The Total amount is **15** liters, and the key ingredient is acid. So, **30%** of **15** liters = **(0.30)(15 liters) = 4.5** liters is pure acid. (b) If we add **3** liters of pure acid, we now have **15+3 = 18** liters of solution of which **4.5+3 = 7.5** liters is pure acid. Therefore, the percentage of acid is now $\frac{\mathbf{7.5}}{\mathbf{18}} = \mathbf{0.41\bar{6}} = \mathbf{41.\bar{6}\%}$.
∵

Example 2: How much of a copper-zinc alloy can you make if it is to be **25%** copper and you have only **60** pounds of zinc available (but unlimited copper)?

Solution: Here we are given specific data about zinc (**60** lbs available), so let's make it the Key ingredient. Thus, we are dealing with a **75%** zinc alloy. Let ***T*** = the total number of pounds of alloy we can make. Then **75%** of ***T*** lbs = **0.75*T*** lbs must be the **60** lbs of zinc available: **0.75*T* = 60**. Solving for ***T*** gives **80** pounds. ∵

Sometimes it is useful to visualize magic containers of the mixture in which the two ingredients are separated with the Key ingredient concentrated at the bottom and the Rest at the top. The pictures for Example 1(a) and Example 2 might look like:



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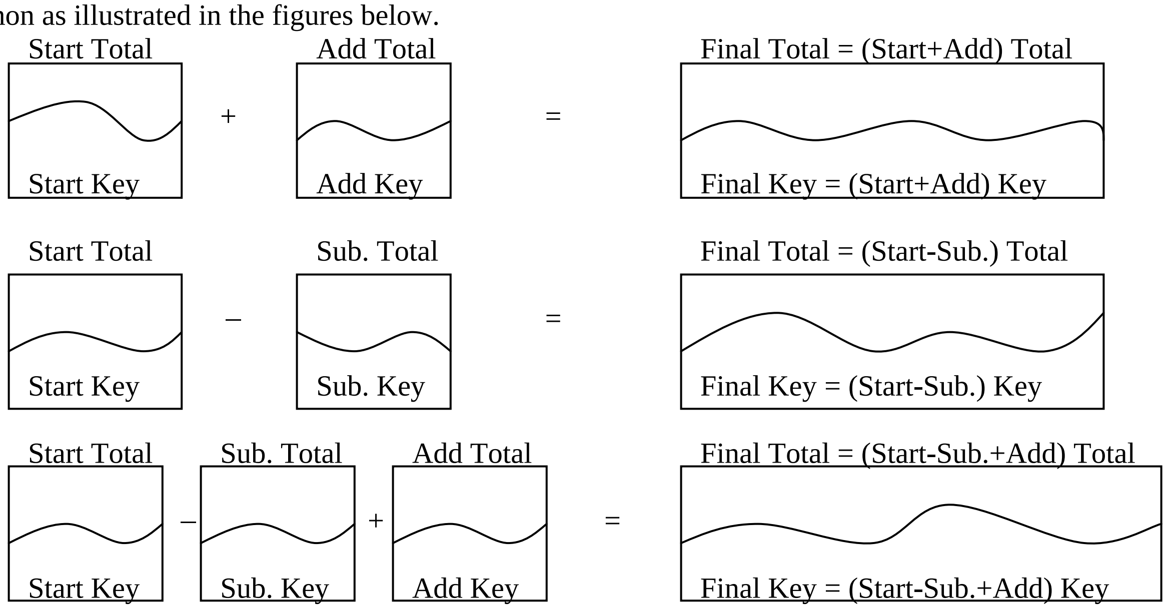
Mixture problems involving these ideas come in a variety of types, but all have a few elements in common as illustrated in the figures below.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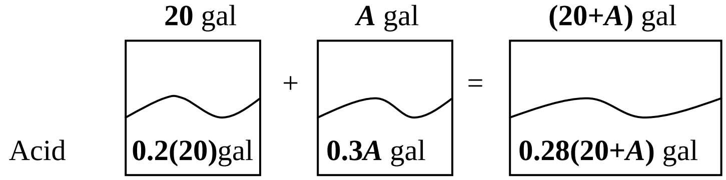
Example 3: Set up the equation to determine how much of a **30%** acid solution must be added to **20** gallons of a **20%** acid solution to yield a **28%** acid solution.

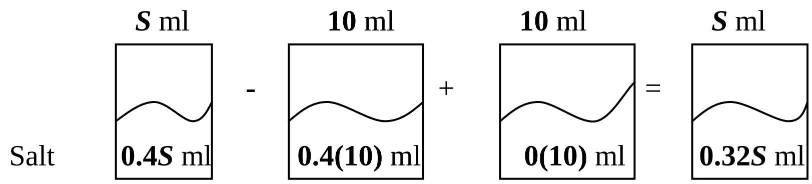
Solution: Let ***A* =** number of gallons of the **30%** acid solution to add. So:



and the sum of the Start and Add Keys = the Final Key: **0.2(20) + 0.3*A* = 0.28(20+*A*)**. (Note: Adding **20** and ***A*** above is legal since they are both measured in gallons.) ∵

Example 4: Set up the equation to determine how much of a **40%** brine solution we started with if we drained **10** milliliters of it and replaced that with **10** milliliters of pure water and ended up with a **32%** brine solution.

Solution: Let ***S*** = number of milliliters of the **40%** solution with which we started.



Notice here that water is **0%** salt and hence the **0(10)** ml for the Salt in the third container. Keeping track of salt gives: **0.4*S* – 0.4(10) + 0(10) = 0.32*S***. ∵

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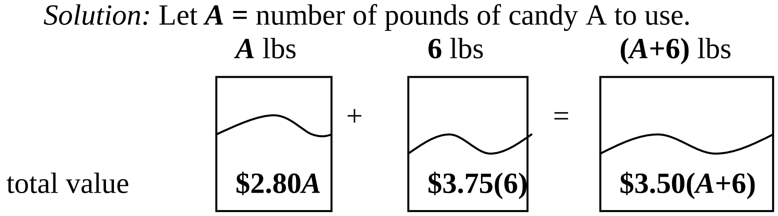
Value Type:

Value Type mixture problems deal with quantities that have value, usually monetary. We keep track of the ***total value*** just as we keep track of the Key ingredient above. To do that we need to distinguish between value per quantity and total value. For example, if candy A sells for **$2.80** a pound, the "**$2.80** a pound" is a value per quantity statement: $\frac{\mathbf{\$2.80}}{\mathbf{1}\ \boldsymbol{pound}}$; it is NOT total value. A total value would be obtained by multiplying this value/quantity ratio by a specific quantity. For example, the total value of **5** pounds of this candy would be: $\frac{\mathbf{\$2.80}}{\mathbf{1}\ \boldsymbol{pound}} \cdot \mathbf{5}\ \boldsymbol{pounds} = \mathbf{\$14.00}$. Notice the units in this result are dollars.

Example 5: Set up the equation to determine how much candy A worth **$2.80** a pound must we mix with **6** pounds of candy B worth **$3.75** a pound to create a mixture that is worth **$3.50** a pound?

Solution: Let ***A* =** number of pounds of candy A to use.



Visualizing total value in the Key ingredient region of the picture, the products here make sense when looking at the units involved. For example **2.80*A*** stands for $\frac{\mathbf{\$2.80}}{\mathbf{1}\ \boldsymbol{pound}} \cdot \frac{\boldsymbol{A\ pounds}}{\mathbf{1}} = \mathbf{2.80}\boldsymbol{A\ dollars}$. We also note that adding ***A*+6** in the last container is legal since the units for both ***A*** and **6** are pounds. Keeping track of the total value gives the equation: **2.80*A* + 3.75(6) = 3.50(*A*+6)**. ∵
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Problems: (In 3-11 **set up** (do NOT solve) the equation needed to find the requested quantity.)

1. Suppose **3** gallons of a **24%** acid solution is added to **5** gallons of a **40%** acid solution. (a) How much pure acid is in the resulting mixture? (b) What percent of the new mixture is NOT acid?

2. A copper-tin alloy is to be **35%** copper (the rest is tin). How many kilograms of the alloy can you make if you have ***T*** grams of tin?

3. How many gallons of a **50%** acid solution must be mixed with **80** gallons of a **20%** acid solution to get a **40%** acid solution?

4. How many liters of a **25%** alcohol solution must be added to **8** liters of a **40%** alcohol solution to get a **30%** alcohol solution?

5. A zinc-copper alloy is **80%** copper. How many kilograms of this alloy must be mixed with **8** kg of zinc to make an alloy that is **70%** copper?

6. How many barrels of ink worth **$100** per barrel should be mixed with **30** barrels of ink worth **$60** per barrel to get ink worth **$75** per barrel?

7. How many gallons of milk that is **2%** butterfat must be mixed with milk that is **3.5%** butterfat to get **10** gallons of milk that is **3%** butterfat?

8. How much of **20** liters of a **10%** acid solution should be removed and replaced by pure acid to yield a **15%** acid solution?

9. How much water should be added to **20** ounces of a **4%** adoxidil solution to dilute it to a **3%** adoxidil solution?

10. How much pure alcohol should be added to **50** liters of a **60%** alcohol solution to get a **75%** alcohol solution?

11. A full brine tank currently holds **80** liters of a **30%** salt solution. How much of this should be drained and replaced with a **70%** salt solution to get a full tank of a **45%** salt solution?

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1.(a) **2.72** gallons; (b) **66%**

2. $\frac{T}{650}$ kg

3. f = # gallons of **50%** solution: $0.5f + 0.2(80) = 0.4(80 + f)$

4. t = # liters of **25%** solution: $0.25t + 0.4(8) = 0.3(8 + t)$

5. a = # kg of the **80%** copper alloy: $0.8a + 0 = 0.7(a + 8)$

6. h = # barrels of **$100** per barrel ink: $100h + 60(30) = 75(h + 30)$

7. t = # gallons of **2%** butterfat milk: $0.02t + 0.035(10 - t) = 0.03(10)$

8. t = # liters of **10%** acid solution to remove: $0.1(20) - 0.1t + t = 0.15(20)$

9. w = # ounces of water to add: $0.04(20) + 0 = 0.03(20 + w)$

10. a = # liters of pure alcohol to add: $a + 0.6(50) = 0.75(50 + a)$

11. d = # liters to drain and replace: $0.3(80) - 0.3d + 0.7d = 0.45(80)$