Perfect Square Rule

(1) In order to continue the development of quadratic equations we must pause a bit to review basic square roots. We define the (**principal**) **square root** of a non-negative number as follows.

$$\text{For } \ p \geq 0, \ \sqrt{p} = q \ \text{ where } \ q \geq 0 \text{ and } q^2 = p.$$

The symbol $\sqrt{}$ is called a **radical**, and whatever is inside of it is called the **radicand**. Notice that $\sqrt{p}$ is only defined if p is non-negative, and the value of $\sqrt{p}$ is also always non-negative. For example, even though $3^2 = 9$ and $(-3)^2 = 9$, the definition of square root forces $\sqrt{9}$ to be 3. To generate -3 we would have to use $-\sqrt{9}$. Also, note that $\sqrt{0} = 0$ and $\sqrt{1} = 1$.

(2) We have the following rules which help us simplify square roots.

$$\text{If } a, b \geq 0, \text{ then } \sqrt{ab} = \sqrt{a}\sqrt{b}.$$

$$\text{If } a \geq 0 \text{ and } b > 0, \text{ then } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

For example:

(a) $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

(b) $\sqrt{450} = \sqrt{9 \cdot 50} = \sqrt{9 \cdot 25 \cdot 2} = \sqrt{9}\sqrt{25}\sqrt{2} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$

(c) $\sqrt{\frac{33}{4}} = \frac{\sqrt{33}}{\sqrt{4}} = \frac{\sqrt{33}}{2}$

We always simplify square roots if possible.

THERE IS NO RULE FOR SEPARATING THE SQUARE ROOT OF A SUM

$$\sqrt{A + B} \neq \sqrt{A} + \sqrt{B}$$

To convince yourself of this, consider $\sqrt{25}$ which equals 5. But, if we write **25** as **16 + 9**, then there is a temptation to write

$$5 = \sqrt{25} = \sqrt{16 + 9} \ \textcircled{=} \ \sqrt{16} + \sqrt{9} = 4 + 3 = 7 \qquad !!!!!!!!$$

This "equality" is wrong. The square root of a sum is NOT the sum of the square roots.

(3) Square roots come up very naturally in solving some types of quadratic equations. One reason is the following.

Perfect Square Rule:

$$\text{If } q^2 = p, \text{ then } q = \pm\sqrt{p}.$$

Recall the symbol "$\pm A$" is mathematical shorthand for "$+A$ or $-A$". So if $q^2 = p$, then $q = \sqrt{p}$ or $q = -\sqrt{p}$. We use this to solve quadratic equations of the type "*square of a variable thing equals a number*." For example:

(a) $(x+2)^2 - 16 = 0$ isolate the square

$(x+2)^2 = 16$ use Perfect Square Rule

$x + 2 = \pm\sqrt{16} = \pm 4$ subtract 2 from each side

$x = -2 \pm 4$ expand and then simplify

$x = -2 + 4$ or $-2 - 4$

$x = 2$ or -6

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(b) $3(x-3)^2 - 15 = 0$ isolate the square

$3(x-3)^2 = 15$

$(x-3)^2 = 5$ use Perfect Square Rule

$x - 3 = \pm\sqrt{5}$ add 3 to each side

$x = 3 \pm \sqrt{5}$

In example (a) we wouldn't leave our answer as $x = -2 \pm 4$. It is basic arithmetic to find the exact values of x which we did. On the other hand, if a <u>simplified</u> radical appears, then it is OK to leave the $\pm$ in the answer as was done in example (b). It is, however, legal to separate the two answers and write $x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$, and there are future situations when you will do this. But for now, there is no need to take the extra time to do so.

Square roots you **must** know:

$\sqrt{0} = 0$	$\sqrt{1} = 1$	$\sqrt{4} = 2$	$\sqrt{9} = 3$
$\sqrt{16} = 4$	$\sqrt{25} = 5$	$\sqrt{36} = 6$	$\sqrt{49} = 7$
$\sqrt{64} = 8$	$\sqrt{81} = 9$	$\sqrt{100} = 10$	$\sqrt{121} = 11$
$\sqrt{144} = 12$	$\sqrt{169} = 13$	$\sqrt{196} = 14$	$\sqrt{225} = 15$