Linear Equations in One Variable – Mechanics

(1) We should begin the process of solving equations by getting into the habit of asking ourselves some basic questions when faced with any problem. First, "what kind of problem is it?" For now that means "is it an equation or not?" We need to know this in order to fully understand our goal and to select the proper method of attack. For example, $\mathbf{6x - 3(5x + 2) + 4 - 5x}$ is not an equation and our goal is to simplify (answer: $\mathbf{2 - 14x}$); but $\mathbf{6x - 3(5x + 2) = 4 - 5x}$ is an equation and our goal is to find the value(s) of $\boldsymbol{x}$ which make the equation true (i.e., make the left side equal the right side). Once we determine that we have an equation, the next question is "what type of equation is it?" First of all, if both sides are polynomials, then it is a **polynomial equation**. For example, $\sqrt{\mathbf{x + 3}} = \mathbf{x - 2}$ is NOT a polynomial equation, but $\mathbf{(x + 3)^2 = x - 2}$ is. If it is a polynomial equation, we classify it by the higher of the two degrees. For example, $\mathbf{3x - 2 = 5}$ and $\mathbf{4 - 2x = 3x + 7}$ are **linear equations**; while $\mathbf{4x + 2 = x^2 - 3x}$, $\mathbf{3x^2 + 5x = 2}$ and $\mathbf{x^2 + 4 = 2x^2 - 3x}$ are **quadratic equations**.

(2) For a quick definition: a **linear equation** is a polynomial equation in which the highest power on the variable is one. **It is assumed that you can solve any linear equation including literal equations**. A **literal equation** is an equation involving variables other than the one for which you must solve. We'll see an example of solving one below.

(3) What follows are examples of solving linear equations using the **SCCC** (pronounced "sick") method: **Simplify, Condense, Constant, Coefficient** As with many types of problems in mathematics, there are many ways of solving linear equations. If you have developed a method that consistently works for you, then stick with it. However, if you cannot consistently come up with the correct answer, I suggest you try the SCCC method – it is guaranteed to work unless you make an algebra error along the way. The first example should be very simple for you, but I'm going to do one part in a very meticulous way since it is that part which causes many students trouble when doing literal equations.

Example 1: Solve $\mathbf{6x - 3(5x - 2) = 4 - 5x}$

Solution:

Simplify	$\mathbf{6x - 15x + 6 = 4 - 5x}$	Remove grouping symbols by using the distributive law.
	$\mathbf{-9x + 6 = 4 - 5x}$	Combine like terms on each side as needed.
Condense	$\mathbf{6 = 4 - 5x + 9x}$	Using addition or subtraction, as appropriate, put all $\boldsymbol{x}$ terms on one side. I chose the right side to make the coefficient on $\boldsymbol{x}$ positive.
	$\mathbf{6 = 4 + (-5 + 9)x}$	**Use distributive law to create a single x term.** *This is the step I'm being very meticulous about.*
	$\mathbf{6 = 4 + 4x}$	
Constant	$\mathbf{6 - 4 = 4x}$	Using addition or subtraction, as appropriate, put all the non-$\boldsymbol{x}$ terms (*constants*) on the other side.
	$\mathbf{2 = 4x}$	
Coefficient	$\mathbf{\frac{2}{4} = x}$	Divide both sides by the coefficient of $\boldsymbol{x}$ to isolate $\boldsymbol{x}$.
	$\mathbf{x = \frac{1}{2}}$ ∵	

It's the use of the distributive law in the middle of the condense step that causes many students trouble in literal equations. Most students see $\mathbf{-5x + 9x}$ and automatically think "combine like terms" and write $\mathbf{+4x}$ without realizing why that is legal. The "why" is the distributive law: $\mathbf{-5x + 9x = (-5 + 9)x = 4x}$. **The entire concept of "combining like terms" is really a straight forward application of the distributive law.**

Example 2: Solve for x in $6x - y(5x - a) = c - bx$

Solution: Can you see how this looks identical to the first equation except we have letters in place of some of the numbers? This is a **literal equation** so we have to specifically be told what letter to solve for (x in this case). What kind of an equation in x is it? Linear. So we use the SCCC method.

Simplify	$6x - 5yx + ay = c - bx$	Remove grouping symbols by using the distributive law.
Condense	$6x - 5yx + bx + ay = c$	Using addition or subtraction, as appropriate, put all x terms on one side. I chose the left side since there were fewer terms to deal with that way.
	$(6 - 5y + b)x + ay = c$	**This is the key step – use the distributive law to create a single x term.**
Constant	$(6 - 5y + b)x = c - ay$	Using addition or subtraction, as appropriate, put all the non-x terms (*constants*) on the other side.
Coefficient	$x = \dfrac{c - ay}{6 - 5y + b} \because$	Divide both sides by the coefficient of x to isolate x.

As you saw above, the key action was using the distributive law in the second part of the Condense step. Note also that any letter other than the one we're solving for is considered a *constant* in this procedure.

(4) Remember that if you're solving any equation for a particular variable, then that variable cannot appear in your answer. So, if you ever end up with something like $x = \dfrac{a - x}{b}$, you know you are not done!

(5) Recall that a **ratio** is a quotient. A statement that says two ratios are equal is called a **proportion**.

A proportion, $\dfrac{a}{b} = \dfrac{c}{d}$, is true if and only if the cross products are equal: $ad = bc$.

{Proof: $\dfrac{a}{b} = \dfrac{c}{d} \Leftrightarrow \dfrac{a}{b} - \dfrac{c}{d} = 0 \Leftrightarrow \dfrac{ad - bc}{bd} = 0 \Leftrightarrow^{*} ad - bc = 0 \Leftrightarrow ad = bc$; where the $\Leftrightarrow^{*}$ justification comes from the fact that a quotient with a non-zero denominator is **0** if and only if the numerator is **0**.}

Example 3: Solve $y = \dfrac{x + 2}{x - a}$ for x.

Solution: Since $y = \dfrac{y}{1}$, this is a proportion: $\dfrac{y}{1} = \dfrac{x + 2}{x - a}$. So equate the cross products: $y(x - a) = 1(x + 2)$ and solve. This is a linear equation, so we use SCCC.

Simplify	$yx - ya = x + 2$	(remove the parentheses)
Condense	$yx - x - ya = 2$	(put x terms to one side)
	$(y - 1)x - ya = 2$	(**use distributive law to get a single x term**)
Constant	$(y - 1)x = 2 + ya$	(we use addition here)
Coefficient	$x = \dfrac{2 + ya}{y - 1} \because$	(division used to make the x coefficient a 1)