MAT 105 – Regular Test #3
Sample #6

RT #3 typically includes problems like the following along with DRT, mixture, and piecewise function problems (which are not sampled here – see The Fundamentals). There may be other problems which are not exemplified here, so be sure you study all the ideas covered in your section of the course. The ANSWERS are found at the end of this document.

See other Samples for more practice.

1. Complete the square and solve for *x*: $3x^2 + 5x - 1 = 0$.
2. Write an equation for a parabola that has vertex at $(0,0)$.
3. Solve for *x* algebraically: (i) $6^{4x-1} = 36^{3-2x}$; (ii) $x^{\frac{3}{5}} = -8$
4. (Round to two decimal places.) A population quadruples every **7** hours. There are **3** now, how long until there are **25,500**?
5. Solve for *x* algebraically: $5 = \sqrt{3x+13} - x$
6. If *c* is a negative number, determine if the following are positive, negative, **0**, or do not exist as real numbers (DNE): (i) $-c^{\frac{6}{5}}$; (ii) $c^{-\frac{3}{8}}$; (iii) $(-c)^{-\frac{5}{2}}$; (iv) $-c^{-\frac{5}{8}}$
7. (Round the percent to two decimal places.) A population grew from **6** to **1420** in **6** days. What was the average daily rate of growth?
8. Rationalize the denominator: (i) $\dfrac{1}{\sqrt{3}+\sqrt{7}}$; (ii) $\dfrac{1}{\sqrt[4]{x^8y^9z^{10}}}$; (iii) $\dfrac{1}{\sqrt[5]{x^8y^9z^{10}}}$
9. An object is propelled into the air. The height *h*, in meters, that it is above the ground at any time *t* minutes after being released is given by: $h = 22 + 7t - t^2$. (i) How long does it stay in the air? (ii) What is its maximum height? (iii) How long after it is released will it be **30.75** meters high for the first time?

BRIEF ANSWERS (Detailed answers are below the line):

1. $\left(X + \frac{5}{6}\right)^2 = \frac{37}{36}$ leads to $x = \dfrac{-5 \pm \sqrt{37}}{6}$

2. For example, $y = x^2$

3. (i) $\frac{7}{8}$; (ii) -32

4. **45.69** hours

5. $-3, -4$

6. (i) neg.; (ii) DNE; (iii) pos.; (iv) DNE

7. **148.71%**

8. (i) $\dfrac{\sqrt{3}-\sqrt{7}}{-4}$; (ii) $\dfrac{\sqrt[4]{y^3z^2}}{x^2y^3z^3}$; (iii) $\dfrac{\sqrt[5]{x^2y}}{x^2y^2z^2}$

9. (i) **9.352** minutes; (ii) **34.25** meters; (iii) **1.629** minutes

DETAILED ANSWERS:

1. Remember to make monic first, i.e., the leading coefficient must be **1:**

$$3x^2 + 5x - 1 = 0$$

$$x^2 + \frac{5}{3}x \qquad = \frac{1}{3}$$

$$\frac{1}{2}\left(\frac{5}{3}\right) = \frac{5}{6}$$

$$x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \frac{1}{3} + \left(\frac{5}{6}\right)^2 = \frac{1}{3} + \frac{25}{36}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{37}{36}$$

$$x + \frac{5}{6} = \pm\sqrt{\frac{37}{36}} = \pm\frac{\sqrt{37}}{6}$$

$$x = -\frac{5}{6} \pm \frac{\sqrt{37}}{6} = \frac{-5 \pm \sqrt{37}}{6}$$

2. $y = (x - 0)^2 = x^2$

3. (i) $6^{4x-1} = 36^{3-2x}$

$$6^{4x-1} = (6^2)^{3-2x}$$

$$6^{4x-1} = 6^{2(3-2x)}$$

$$4x - 1 = 6 - 4x$$

$$8x = 7$$

$$x = \frac{7}{8}$$

(ii) $x^{\frac{3}{5}} = -8$

$$\left(x^{\frac{3}{5}}\right)^{\frac{5}{3}} = (-8)^{\frac{5}{3}}$$

$$x = \left(\sqrt[3]{-8}\right)^5$$

$$x = (-2)^5$$

$$x = -32$$

4. Let ***H*** be the number of hours and ***p*** be the population. Fill in the data starting with ***H*** and ***p*** and add the extra column after that.

$\frac{H}{7}$	H	p
0	0	3
1	7	$4 \cdot 3$
2	14	$4(4 \cdot 3) = 4^2 \cdot 3$
3	21	$4(4^2 \cdot 3) = 4^3 \cdot 3$
4	28	$4^4 \cdot 3$

$$p = 3 \cdot 4^{\frac{H}{7}}$$

So, $25,500 = 3 \cdot 4^{\frac{H}{7}}$

$$\frac{25,500}{3} = 4^{\frac{H}{7}}$$

(continued on the next page)

Graph $Y1 = 4 \wedge (X \div 7)$ and $Y2 = 25500 \div 3$. A window of $X_{\min} = 0,\ X_{\max} = 55,\ X_{scl} = 3,\ Y_{\min} = 0,\ Y_{\max} = 10000,\ Y_{scl} = 1000$ gives a good viewing rectangle. Using 2nd/CALC/5:intersect gives the number of hours (**X** value) to be ≈ 45.69.

5. This is a radical equation. So: Isolate a radical, Square both sides, Solve the resulting equation, and finally Check your proposed answers (in order to throw out any extraneous roots): (The ISSC process.)

$$5 = \sqrt{3x+13} - x$$
$$x + 5 = \sqrt{3x+13}$$
$$(x+5)^2 = (\sqrt{3x+13})^2$$
$$x^2 + 10x + 25 = 3x + 13$$
$$x^2 + 7x + 12 = 0$$
$$(x+3)(x+4) = 0$$
$$x = -3 \text{ or } x = -4$$

Check:

$x = -4$	$x = -3$
$5 \overset{?}{=} \sqrt{3(-4)+13} - (-4)$	$5 \overset{?}{=} \sqrt{3(-3)+13} - (-3)$
$5 \overset{?}{=} \sqrt{1} + 4$	$5 \overset{?}{=} \sqrt{4} + 3$
$5 \overset{?}{=} 1 + 4$	$5 \overset{?}{=} 2 + 3$
$5 = 5$ ***OK***	$5 = 5$ ***OK***

So, $x = -3$ and $x = -4$ both work.

6. c is negative

(i) $-c^{\frac{6}{5}} = -\left(\sqrt[5]{c}\right)^6$: 5th root of a neg. is neg.; 6th power of a neg. is positive; negation of a positive is negative. So, Neg.

(ii) $c^{-\frac{3}{8}} = \dfrac{1}{c^{\frac{3}{8}}} = \dfrac{1}{\left(\sqrt[8]{c}\right)^3}$: 8th root of a negative number is not real, so ***DNE***.

(iii) $(-c)^{-\frac{5}{2}} = \dfrac{1}{(-c)^{\frac{5}{2}}} = \dfrac{1}{\left(\sqrt{-c}\right)^5}$: The negation of a neg. is positive; the square root of a positive is positive; the 5th power of a positive is positive; **1** divided by a positive number is positive. So, Pos.

(iv) $-c^{-\frac{5}{8}} = \dfrac{-1}{c^{\frac{5}{8}}} = \dfrac{-1}{\left(\sqrt[8]{c}\right)^5}$: the 8th root of a negative number is not real. So, ***DNE***.

7. The average rate of growth formula is $R(1+r)^n = N$. Here $R = 6,\ n = 6$ and $N = 1420$. So we have:

$$\left\{\begin{aligned} &6(1+r)^6 = 1420 \\ &(1+r)^6 = \frac{1420}{6} = \frac{710}{3} \\ &\left((1+r)^6\right)^{\frac{1}{6}} = 1 + r = \left(\frac{710}{3}\right)^{\frac{1}{6}} \approx 2.487078629 \\ &r \approx 1.487078629 \approx 148.71\% \end{aligned}\right\}$$

8. (i) Use the conjugate: $\frac{\mathbf{1}}{\sqrt{\mathbf{3}}+\sqrt{\mathbf{7}}} \bullet \frac{\sqrt{\mathbf{3}}-\sqrt{\mathbf{7}}}{\sqrt{\mathbf{3}}-\sqrt{\mathbf{7}}} = \frac{\sqrt{\mathbf{3}}-\sqrt{\mathbf{7}}}{(\sqrt{\mathbf{3}})^{\mathbf{2}}-(\sqrt{\mathbf{7}})^{\mathbf{2}}} = \frac{\sqrt{\mathbf{3}}-\sqrt{\mathbf{7}}}{\mathbf{3}-\mathbf{7}} = \frac{\sqrt{\mathbf{3}}-\sqrt{\mathbf{7}}}{-\mathbf{4}}$

(ii) To get rid of a 4th root, every quantity in the radicand must be raised to a power that is a multiple of **4**.
So: $\frac{\mathbf{1}}{\sqrt[4]{x^8y^9z^{10}}} \bullet \frac{\sqrt[4]{y^3z^2}}{\sqrt[4]{y^3z^2}} = \frac{\sqrt[4]{y^3z^2}}{\sqrt[4]{x^8y^{12}z^{12}}} = \frac{\sqrt[4]{y^3z^2}}{x^2y^3z^3}$. (iii) To get rid of a 5th root, every quantity in the radicand must be raised to a power that is a multiple of **5**. $\frac{\mathbf{1}}{\sqrt[5]{x^8y^9z^{10}}} \bullet \frac{\sqrt[5]{x^2y}}{\sqrt[5]{x^2y}} = \frac{\sqrt[5]{x^2y}}{\sqrt[5]{x^{10}y^{10}z^{10}}} = \frac{\sqrt[5]{x^2y}}{x^2y^2z^2}$.

9. Place into $\boldsymbol{Y1}$ the expression $\mathbf{22}+\mathbf{7}x-x^{\mathbf{2}}$ and graph. A reasonable window is **Xmin = 0** (since we are only interested in non-negative time), **Xmax = 12, Ymin =** $-\mathbf{10}$ **, Ymax = 50, Yscl = 5**. (i) Using 2nd/CALC/2:zero, do the Left Bound/Right Bound/Guess "thing" with the cursor around where the curve crosses the horizontal axis (we're finding where the height is **0**). This gives a time (**X** value) of **9.352** minutes. (ii) Using 2nd/CALC/4:maximum, do the LB/RB/Guess "thing" with the cursor around where the curve seems to peak. This gives a height (**Y** value) of **34.25** meters. (iii) Go back to the equation editor and enter **30.75** in **Y2**. Graph. Using 2nd/CALC/5:intersect, do the First curve/Second curve/Guess "thing" with the cursor near the left most intersection point when doing the Guess. This gives a time (**X** value) of **1.629** minutes.