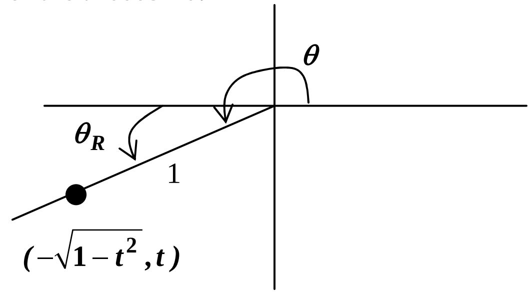
An Inverse Trig Function Problem Type

(1) Given $\sin\theta = t$ and $\theta \in QIII$, write θ in terms of the ***arccosine***.

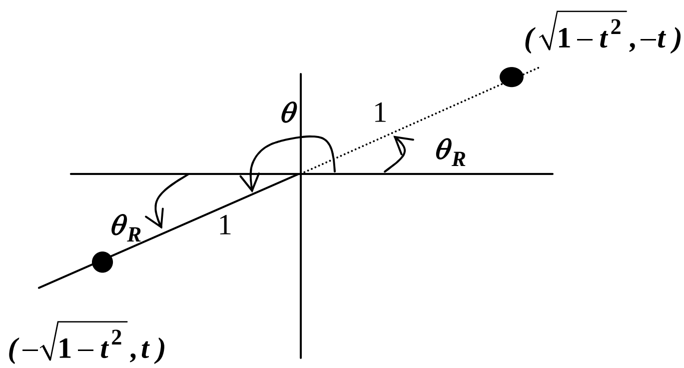
Since $\sin\theta = \frac{t}{1}$ $\left(\frac{y}{r}\right)$, we take $y = t$ and $r = 1$.

From Pythagoras, $x = -\sqrt{1-t^2}$, where the negative comes from the quadrant information.



So $\theta = \pi + \theta_R$

Construct the reference angle in standard position.



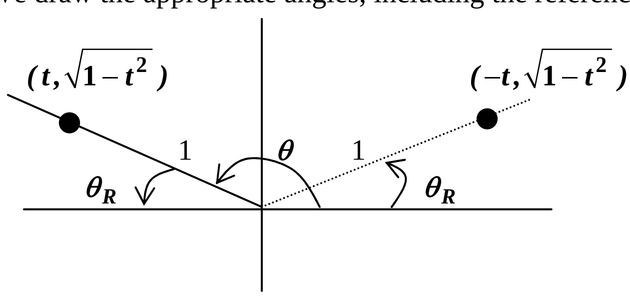
So $\cos\theta_R = \frac{\sqrt{1-t^2}}{1} = \sqrt{1-t^2}$

and hence $\theta_R = \cos^{-1}\sqrt{1-t^2}$

Thus, $\theta = \pi + \theta_R = \pi + \cos^{-1}\sqrt{1-t^2}$

(2) Given $\cos\theta = t$ and $\theta \in QII$, write θ in terms of the ***arctangent***.

We draw the appropriate angles, including the reference angle placed in standard position.



Since $\cos\theta = t = \frac{t}{1}$ $\left(\frac{x}{r}\right)$, we can take $x = t$ and $r = 1$. Pythagoras gives us $y = \sqrt{1-t^2}$.

We also note that $\theta = \pi - \theta_R$.

Now, $\tan\theta_R = \frac{\sqrt{1-t^2}}{-t}$, so $\theta_R = \tan^{-1}\left(\frac{-\sqrt{1-t^2}}{t}\right)$. Hence $\theta = \pi - \tan^{-1}\left(\frac{-\sqrt{1-t^2}}{t}\right)$.

(Note that since t is negative, the quantity $\frac{-\sqrt{1-t^2}}{t}$ is positive, and hence the arctangent of it does indeed give us the reference angle.)